

# OH Emission Spectra of Hybrid Rocket Motors Using PMMA and HTPB

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## Nomenclature

$E_{Total}$	=	total energy of a collection of molecules
$T_{Translational}$	=	translational energy of a collection of molecules
$R_{Rotational}$	=	rotational energy of a collection of molecules
$E_{Vibrational}$	=	vibrational energy of a collection of molecules
$E_{Electronic}$	=	electronic energy of a collection of molecules
$f(?)$	=	Boltzmann of speed of molecules as a function of Kelvin temperature and mass of molecule
$m$	=	mass of a molecule in kg
$k$	=	Boltzmann constant
$T$	=	Kelvin temperature
$?$	=	molecular speed
$N_i$	=	number of molecules occupying energy level $i$
$N_0$	=	number of molecules occupying the lowest energy level 0
$g_i$	=	degeneracy of energy level $i$
$Q(T)$	=	partition function evaluated at temperature $T$
$[A]$	=	concentration of species A in moles per liter
$e^-$	=	electron
$K$	=	equilibrium constant
$?$	=	wavelength of radiation
$?(?)$	=	density of radiation as a function of wavelength and Kelvin temperature
$? ?_{app}$	=	apparatus function which is a measure of the wavelength resolution of a spectrometer
$E_n$	=	energy of level $n$ in joules per molecule
$I_{nm}$	=	intensity of emission from energy level $n$ to energy level $m$
$G(?)$	=	Gaussian distribution with independent variables $?$ and $?$

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## Abstract

Our goal is development of diode laser sensors that monitor hybrid rocket motor emissions in the near-infrared. Quantities desired from the measurements are temperature, molecule number density, pressure and velocity. Targeted species include carbon dioxide, water and OH radicals. The measuring instrumentation will become part of the rocket motor control system to optimize thrust, fuel efficiency and safety response. To validate OH radical information gathered from the near-infrared region using diode lasers, studies are being conducted that measure OH radical emission spectra in the ultraviolet region, 305 to 330 nm that includes the  $A^2\Sigma^+, v = 0 - X^2\Sigma^+, v' = 0$  band. Two compact spectrometers, having 590 grooves/mm and 3600 grooves/mm gratings and 2048 pixel CCD line camera detectors were controlled with software that performed synchronous, repetitive spectral capture. Spectra were acquired by the spectrograph through an armored 400  $\mu\text{m}$  fiber optic cable, 2 m long, terminated with a UV collimating lens pointing towards the rocket exhaust. The lens was mounted in one end of a black Delrin cylinder 100 mm long having a 6 mm bore, allowing investigation of 8 mm<sup>2</sup> segments of the rocket plume. A test stand was designed to allow precise control of experimental parameters. Simple adjustments of the test stand made it possible to rapidly measure different parts of the plume. The design included provision for simultaneous recording of UV, Vis and NIR spectra of the same part of the rocket plume. The exhaust plumes from PMMA, polymethylmethacrylate, fuel grains were mapped spatially and temporally. Several investigators have modeled the spectrum of OH radicals in order to determine the combustion temperatures of different processes. Their methods were employed to characterize hybrid rocket plume emissions. Temperatures estimated for the PMMA fuel grains ranged from 2600 to 3400 K using 5 to 30 psig oxygen.

## I. Introduction

This research the development of sensors for hybrid rocket motors, has as its goal the development of near-infrared diode laser sensors for characterizing hybrid rocket plumes. In order to validate the results of the diode laser measurements, it was necessary to utilize other spectroscopic methods that could provide confirmation of results obtained. Having two sets of measurements that agree using two different methods would provide strong evidence for the validity of the results. Parameters to be measured in hybrid rocket exhaust plumes include water vapor, carbon monoxide, carbon dioxide and hydroxyl radicals. Temperature is a fundamental quantity that must also be determined. Spectra obtained can be used to calculate species concentrations and temperatures, both spatially and temporally, in rocket exhaust plumes. In this paper we report our findings from a study of the emission spectra of hybrid rocket exhaust plumes in the ultraviolet, UV, visible, VIS, and near-infrared, NIR, regions of the electromagnetic spectrum.

After a review of the scientific literature and initial spectral measurements, it became apparent that rocket plume spectroscopy, or any combustion plume spectroscopy, is a formidable task. Most quantitative studies of combustion processes utilize pre-mixed, homogeneous gas flows through flat-field burners. This approach greatly reduces the complexity of the system to be studied and allows for better quantitative conclusions to be drawn. A hybrid rocket plume, on the other hand, originates from the burning of a solid propellant, the fuel grain, with an oxidant, such as oxygen, hydrogen peroxide or nitrous oxide. Unburnt and partially burnt particles of the solid propellant are projected into the plume where they produce a brilliantly colored flame that emits radiation over most of the visible and a portion of the near-infrared spectral range. The interior of the exhaust plume reaches very high temperatures. However, the axial temperature gradient of the plume is quite large and temperature falls toward ambient temperatures over relatively short distances. This rapid temperature decrease is responsible for spectral self-absorption when radiant energy emitted by hot molecules in the interior of the plume is absorbed by cooler molecules of the same type in the cooler edges of the plume. Another complexity encountered is that spectral measurements of exhaust plumes are averaged over the cone of view encompassed by the detector. This averaging applies to both the concentrations of the molecules and temperatures. Nevertheless, important conclusions and

understanding of rocket motor behavior can be realized that leads to more powerful, safer, better controlled and more environmentally friendly motor designs.

It is important, at the outset, to have a good understanding of the concept of temperature. Temperature is typically defined thermodynamically by means of the Carnot cycle. Temperature is defined in the kinetic theory of gases as proportional to the average translational energy of a collection of gas molecules. In spectroscopy, temperature is defined through the Planck Blackbody Radiation Formula<sup>1</sup>. These three approaches yield the same value for temperature only if the system under consideration is in a state of thermodynamic equilibrium. To define thermodynamic equilibrium, consider the total energy of a large number of molecules. The total energy can be expressed as the sum of the different kinds of energy possessed by these molecules as shown in Eq. (1).

$$E_{Total} = E_{Translational} + E_{Rotational} + E_{Vibrational} + E_{Electronic} \quad (1)$$

Five requirements must be met to have thermodynamic equilibrium<sup>2</sup>:

1. The molecules must have a translational distribution of molecular velocities according to the Maxwell-Boltzmann distribution of molecular velocities expressed in Eq. (2).

$$f(v) = 4pv^2 \left( \frac{m}{2pkT} \right)^{3/2} e^{-\frac{mv^2}{2kT}} dv \quad (2)$$

2. The molecules must be distributed over the allowed range of rotational, vibrational and electronic energy states according to the Boltzmann distribution of energies given by Eq. (3).

$$\frac{N_i}{N_0} = \frac{g_i}{Q} e^{-\epsilon_i/kT} \quad (3)$$

3. The ionization of the molecules must obey the Saha equation stated in Eq. (4).

$$K_i = \frac{[M^+][e^-]}{[M]} \quad (4)$$

4. Molecular dissociation must obey the law of mass action as illustrated in Eq. (5).

$$AB \leftrightarrow A + B$$

$$K_d = \frac{[A][B]}{[AB]} \quad (5)$$

5. The radiation density must obey Planck's radiation formula given by Eq. (6).

$$\rho(\nu) d\nu = \frac{8\pi h c}{\nu^3} \frac{1}{e^{hc/\nu kT} - 1} d\nu \quad (6)$$

Thermodynamic equilibrium would be difficult to achieve in a rocket motor exhaust plume. Exhaust plumes radiate energy to their surroundings. They blast off material into and entrap molecules from their environment. Some of their energy is dissipated in the acoustic region. Also, there is only a short time available for equilibrations and relaxations required for a state of equilibrium.

Although exhaust plumes, as a whole, may not exhibit thermodynamic equilibrium, it is reasonable to expect that small regions of the plume may have local states of thermal equilibrium, LTE, that would have a specified

temperature, or that there would be regions of the plume in which one or more of the five criteria mentioned above would be closely approximated. In this latter case a "rotational" or "vibrational" or "electronic" temperature could be estimated from the respective Boltzmann distribution equations for rotational, vibrational or electronic energy states. As an example of this method, a "rotational" temperature could be estimated by fitting the rotational spectrum of a molecule obtained from an exhaust plume to Eq. 3 using temperature as the independent variable. This would result in a "rotational" temperature. It is very important not to confuse these specialized temperatures with the true thermodynamic temperature. Therefore, these temperatures are named by giving the method used to determine them, such as the vibrational temperature or the electronic temperature. The extent to which these different temperatures agree is a measure of how close a local portion of the exhaust plume is to thermodynamic equilibrium and to possessing the thermodynamic temperature. Of the different specialized temperatures, translational temperature is closest to the true temperature since translational energy is equipartitioned in less than 10 collisions in approximately  $10^{-8}$  seconds at 1 bar pressure<sup>2</sup>. Rotational states require approximately  $10^3$  collisions for relaxation to equilibrium while vibrational relaxation requires about  $10^5$ . Electronic excitations and dissociations need approximately  $10^7$  collisions. In this study, the temperature of hybrid rocket plumes will be estimated using spectra obtained in the ultraviolet and visible spectrum.

## II. Experimental

Two spectrometers were used for data collection. A UV/VIS spectrometer, StellarNet, Inc Model EPP2000C-UV-VIS, covering the range from 190 to 850 nm was purchased. This spectrometer has a 40 mm concave, 590 grooves per mm, grating that needs no mirrors, thereby minimizing energy losses due to reflection and light scattering. With the installed 25  $\mu\text{m}$  slits, the spectrometer's spectral resolution is 0.85 nm and has as its detector a 2048 pixel CCD linear array with a pixel size of 14  $\mu\text{m}$  by 200  $\mu\text{m}$ . The supplied instrument software, SpectraWiz Vers. 4.0d, in combination with a built in 12 bit digitizer, allows episodic captures with detector integration times ranging from 2 ms to 65 s. Spectral energy is transmitted to the monochromator through a 400  $\mu\text{m}$  by 4 meter UV-VIS-SR armored fiber optic cable. The fiber is terminated with a quartz Lens-Q-Col collimating lens from StellarNet, Inc. The unit measures 7 cm x 10.8 cm x 15.5 cm. The second spectrometer, covering the wavelength range 240 to 340 nm, was a special order spectrometer from StellarNet, Inc. This instrument, Model EPP2000-UV5-HR, has a 3600 grooves per mm grating and an entrance slit width of 25  $\mu\text{m}$ . A 2 m F400-UV-VIS-SR fiber optic cable, supplied by StellarNet, Inc. and terminated with a quartz, Lens-Q-Col, collimating lens was employed to gather the radiant energy from the rocket plumes. This spectrometer measures 5.8 cm x 17.3 cm x 20.5 cm. Both of the StellarNet instruments are compact and communicate with computers by means of USB-2 buses. Both spectrometers were calibrated for intensity with standards traceable to NIST. These rugged spectrometers, having no moving parts, are suitable for field studies of hybrid rocket motor exhaust plumes.

Figure 1 is a photograph of the test stand used for the hybrid rocket exhaust plume measurements. A polymethylmethacrylate (PMMA) fuel grain is mounted in a slide that can be adjusted accurately so that spectra of the exhaust plume can be measured reproducibly starting from the end of the fuel grain outwards. A rack and pinion movement adjusts the distance. A ruler is screwed to the base to provide distance information. Oxygen is supplied to the fuel grain by means of a tank of oxygen with a regulator that adjusts the oxygen pressure from 0 to 100 psig.

At right angles to the PMMA fuel grains were mounted two black Delrin cylinders 6.25 mm i.d. by 100 mm long. The fiber optic cables with quartz collimating lenses were screwed into one end while the end facing the exhaust plume was machined to accept 1 in. neutral density filters. These mounting devices were adjustable to accommodate different diameters of exhaust plumes.

## III. Results

Figure 2 shows the results of a typical data capture for a rocket firing. The most noticeable features are the smaller prominent bands around 280 to 300 nm, due to the well documented OH radical emission spectrum, and a large broad emission peak that occurs throughout the visible part of the spectrum and into the near infrared. This large visible peak is due to burning fragments of the fuel grain that have been blasted into the plume. The other noticeable feature is a small peak at about 590 nm. The origins of this peak are unknown. Table 1 lists all the observed spectral features for this PMMA firing.

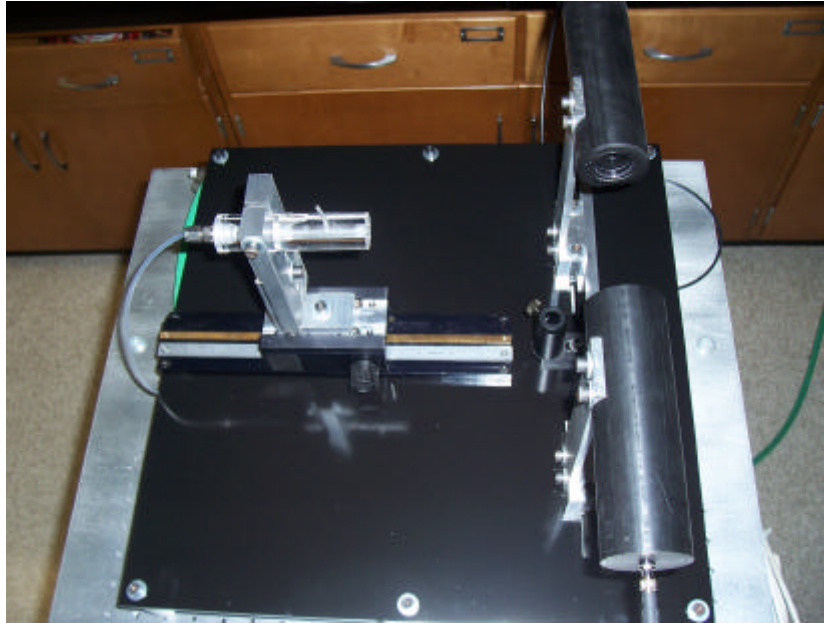


Figure 1. Test stand for hybrid rocket exhaust measurements. Polymethylmethacrylate (PMMA) fuel grains were tested. The oxidant was commercial grade oxygen. Fiber optic cables with quartz collimating lenses transferred the emitted radiation to the two spectrometers. The fiber optic cables were screwed into black Delrin cylinders with a center hole of 6.25 mm diameter by 100 mm long to narrow the field of view. For safety reasons, nozzles were not used for these rocket exhaust studies.

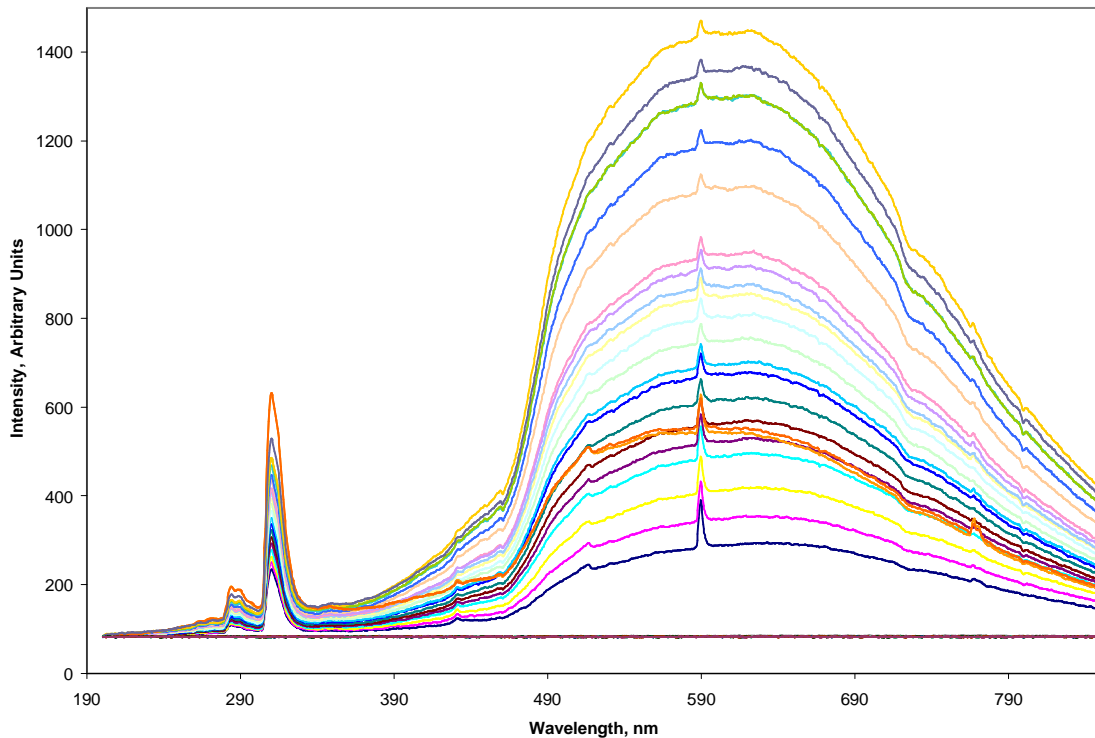
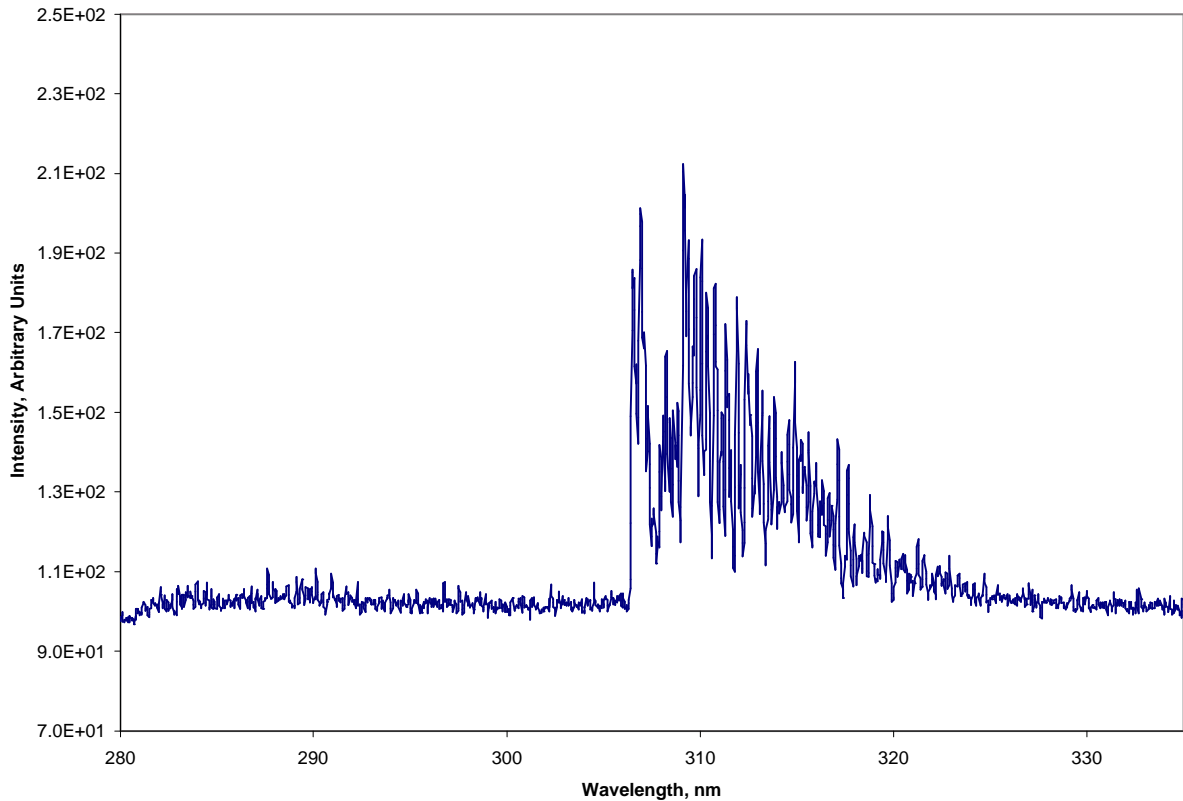


Figure 2. Episodic data capture of PMMA fuel grain, measured 0.0 mm from rocket nozzle with oxygen pressure of 20 psig. Spectra were recorded every 1500 ms with an integration time of 15 ms. Spectra obtained with a StellarNet, Inc. Model EPP2000C-UV-VIS having 25  $\mu\text{m}$  slit width. Grating was a 590 grooves per mm concave holographic grating. The detector was a 2048 pixel CCD detector.

**Table 1. Spectral features observed of exhaust plume of PMMA fuel grain using 20 psig oxygen as oxidant.**

Wavelength	Spectral Feature	Wavelength	Spectral Feature
263.1	Peak	459.1	Shoulder
271.7	Peak	520.5	Peak
283.7	Peak	589.6	Peak
288.3	Peak	736.0	Shoulder
310.2	Peak	767.6	Peak
431.6	Peak	802	Dip



**Figure 3. Spectrum of PMMA rocket plume taken 0.0 mm from the nozzle end of fuel grain. Spectrum captured with StellarNet, Inc. high resolution ultraviolet spectrometer.**

Figure 3 illustrates a single capture spectrum taken with the higher resolution spectrometer optimized for the 280 to 340 nm region in which OH radical emits radiation. Although this band that starts at 306.4 nm looks noisy at first glance, it is quite reproducible and shows all the expected bands for OH at 3000 K. Table 2 compares the spectrum taken with the higher resolution spectrometer with a synthetic spectrum calculated with similar resolution at 3000 K by Pellerin, et al.<sup>3</sup>

**Table 2. Comparison of PMMA Rocket Plume Spectrum with Simulated Spectrum and  $\Delta\lambda_{app} = 0.100$  nm**

Spectral Feature	Wavelength, nm Simulation	Wavelength This Study	Spectral Feature	Wavelength, nm Simulation	Wavelength This Study
Peak 1, I <sub>01</sub>	306.40	306.45	Peak 9, I <sub>24</sub>	308.32	308.35
Shoulder, I <sub>02</sub>	306.52	306.55	Peak 10, I <sub>25</sub>	308.50	308.55
Peak 2, I <sub>03</sub>	306.62	306.60	Shoulder, I <sub>26</sub>	308.64	308.65
Peak 3	306.80	306.85	Peak 11, I <sub>27</sub>	308.72	308.75
Shoulder, I <sub>11</sub>	307.00	307.05	Peak 12,	308.98	309.05
Peak 4, I <sub>12</sub>	307.22	307.25	Shoulder	309.11	309.15
Peak 5, I <sub>13</sub>	307.51	307.50	Peak 13	309.27	309.30
Shoulder, I <sub>14</sub>	307.70	307.70	Peak 14	309.51	309.50
Peak 6, I <sub>21</sub>	307.82	307.85	Peak 15	309.65	309.65
Peak 7, I <sub>22</sub>	307.98	308.00	Peak 16	309.89	309.95
Peak 8, I <sub>23</sub>	308.17	308.15	Average Deviation from simulation		+0.03 nm

Table 3 provides intensities of OH spectral lines at selected wavelengths that were used to estimate the PMMA plume temperatures at different oxygen pressures and using the methods of Pellerin, et al. and the calibration curves shown in Figure 2. a, b, c of their paper and an apparatus function of  $\Delta\lambda_{app} = 0.100$ nm. The Figure 2.d calibration curves could not be used because Pellerin, et al. did not identify the wavelength of band I<sub>04</sub> in their paper.

**Table 3. Temperature of PMMA hybrid rocket plume as a function of oxygen pressure using ratios of intensities of spectral lines in the OH spectrum and the calibration curves of Pellerin, et al.<sup>3</sup>**

PSIG	I <sub>01</sub>	I <sub>02</sub>	I <sub>22</sub>	I <sub>24</sub>	I <sub>02</sub> /I <sub>22</sub>	T, K	I <sub>01</sub> /I <sub>24</sub>	T, K	I <sub>01</sub> /I <sub>22</sub>	T, K
5	84.4	58.3	47.0	43.0	1.24	3250	1.96	>6000	1.80	3000
10	90.9	62.9	50.8	48.2	1.24	3250	1.89	>6000	1.79	3000
12.5	82.0	56.0	46.3	40.2	1.21	3000	2.04	>6000	1.77	3000
15	101.4	70.5	58.5	53.2	1.21	3000	1.91	>6000	1.73	3000
17.5	96.6	66.2	54.1	48.6	1.22	3000	1.99	>6000	1.79	3000
20	123.5	86.1	64.8	61.1	1.33	3000	2.02	>6000	1.91	3250
25	124.5	89.6	73.4	64.7	1.22	3000	1.92	>6000	1.70	2900
30	117.8	82.3	67.8	60.6	1.21	3000	1.94	>6000	1.74	2600

Table 4. Uses the temperature estimation method of Pellerin, et al. for systems with very poor resolution, i.e., those systems with  $\Delta\lambda_{app} > 0.100$ nm. The calibration curve used to estimate the temperature is given in Figure 5. In this method, the highest intensity in each of the two spectral regions given in Table is tabulated and the ratio of these intensities is calculated and compared with the calibration curve in the Pellerin, et al. paper. The curve has such a low sensitivity in the region of 3000 K that the uncertainty is on the order of two hundred degrees Kelvin.

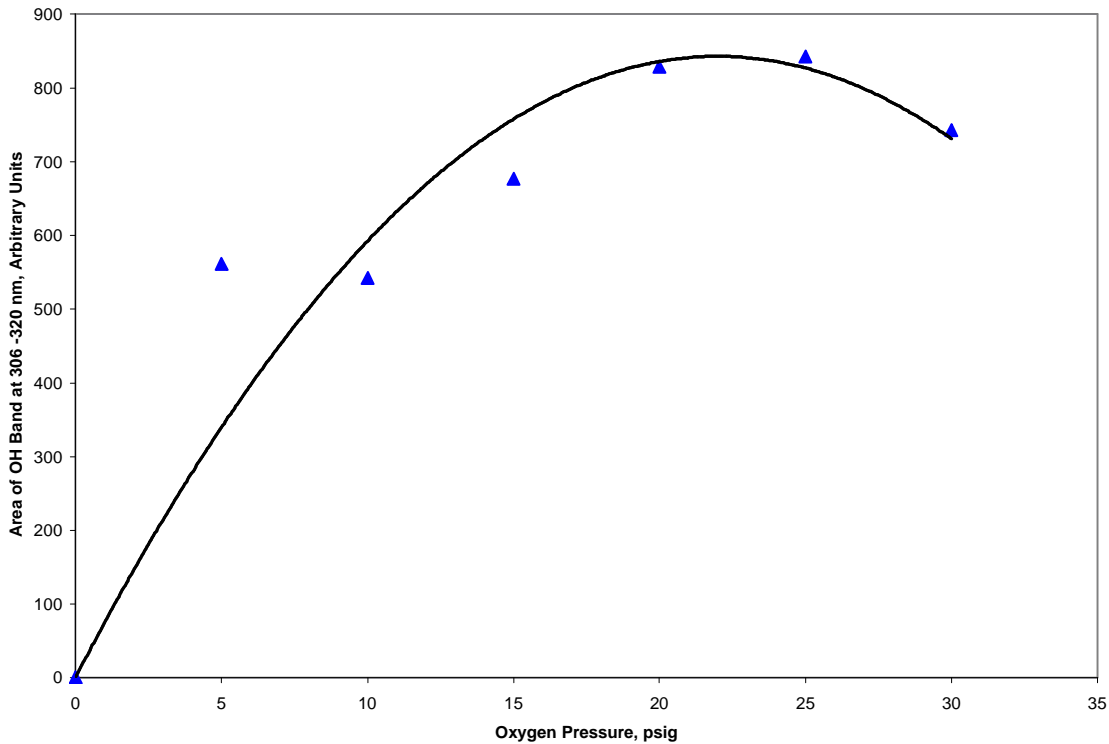
**Table 4. Temperature of PMMA hybrid rocket plumes as a function of oxygen pressure using ratios of maximum intensities of spectral Groups 1 and 2 and the calibration curve, Figure 5, of the paper by Pellerin, et al.<sup>3</sup>**

PSIG	Group 1 306.3-307.3 nm	Group 2 307.6-308.8 nm	Ratio Group 2/Group 1	Temp., K
5	100.9	64.7	0.64	3000
10	108.8	69.2	0.64	3000
12.5	97.7	62.9	0.64	3000
15	123.1	81.4	0.66	3000
17.5	113.4	70.3	0.62	3000
20	148.8	98.6	0.66	3000
25	154.8	98.8	0.64	3000
30	144.4	90.6	0.63	3000

Table 5 summarizes the temperature estimation method of de Izarra using ratios of maximum intensities of spectral groups in the OH spectrum and the calibration data in Tables 2 through 5 in his paper.<sup>4</sup>

**Table 5. Temperature estimation of PMMA hybrid rocket plumes as a function of oxygen pressure using ratios of maximum intensities of Groups  $G_0$ ,  $G_1$  and  $G_{Ref}$  and the data of de Izarra.**

PSIG	$G_0$	$G_1$	$G_{Ref}$	$G_0/G_{Ref}$	Temp., K	$G_1/G_{Ref}$	Temp., K
5	84.4	43.0	108.1	0.7808	3208	0.3978	1636
10	90.9	48.2	117.1	0.7760	3178	0.4116	1690
12.5	82.0	40.2	106.9	0.7671	3120	0.3761	1555
15	101.4	53.2	127.8	0.7934	3299	0.4163	1710
17.5	96.6	48.6	119.8	0.8063	3393	0.4057	1671
20	123.5	61.1	156.5	0.7891	3271	0.3904	1613
25	124.5	64.7	158.7	0.7845	3240	0.4077	1680
30	117.8	60.6	146.8	0.8025	3368	0.4128	1701



**Figure 4. Graph of area under OH radical emission band at 306 – 320 nm as a function of oxygen pressure. Measurements were made with the 3600 grooves per mm spectrometer at a distance of 38 mm from the nozzle.**



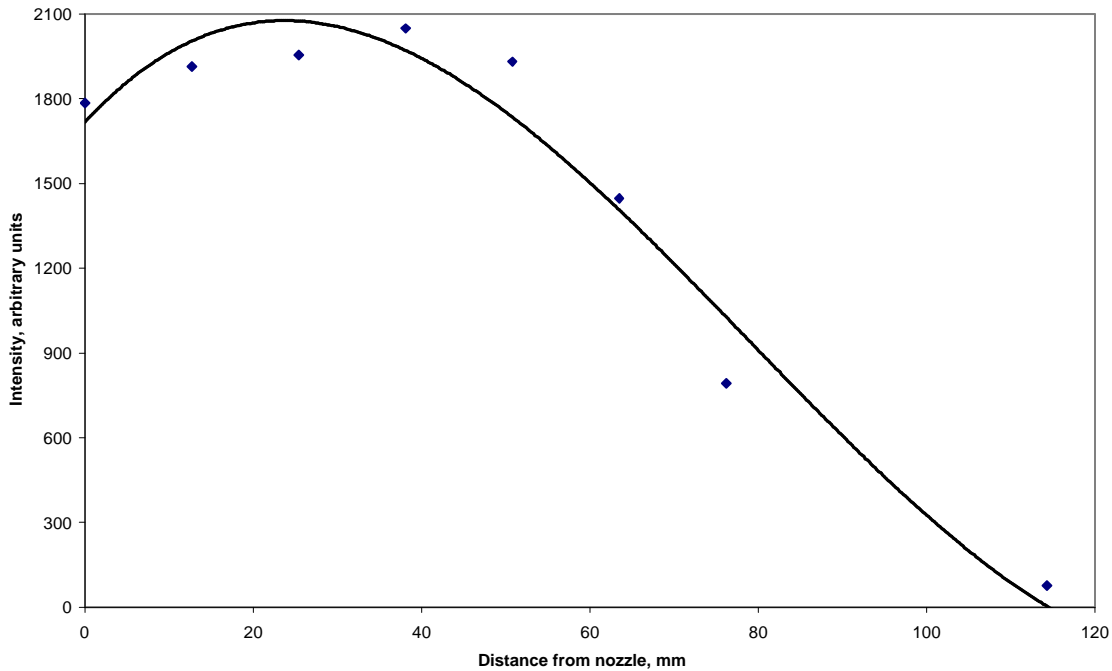


Figure 5. Graph of OH emission intensity of spectral band at 306-320 nm as a function of measurement distance from the exit end of the PMMA fuel grain. The oxygen pressure was maintained at 20 psig.

#### IV. Discussion

Polymethylmethacrylate, PMMA, was chosen for the hybrid rocket fuel grains because of its use as a hybrid rocket fuel, because PMMA contains only C, H and N, as shown in Figure 6, and because of its clean burning

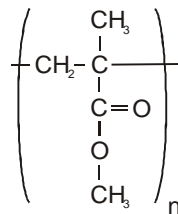


Figure 6. Chemical formula of polymethylmethacrylate, PMMA

characteristics. The spectra in Figure 2 display a very clean spectrum, showing only the OH radical emission in the 280 to 330 nm region. The remaining spectral emissions from 390 to 840 nm, other than the bright emission from burning fuel grain particles show very little evidence of other combustion products. There is never any noticeable soot formed during a burn. When comparing the emission spectrum of PMMA with a typical hydrocarbon emission spectrum, such as the propane – oxygen flame, it is obvious that for PMMA, the emission bands are almost non-existent other than that of OH. There is only the slightest hint of a C-H band in the C-H band region of 420 to 435 nm and none for C-H in the 385 to 400 nm region. Also, the C<sub>2</sub> bands at 460 -470 nm and 490 – 515 nm and 540 – 560 nm are absent.

The "high resolution" StellarNet, Inc. spectrometer does a good job of measuring the UV OH band, considering its small size and simplicity. Pellerin, et al. synthesized a UV OH spectrum using the high resolution spectral data of Dieke and Crosswhite measured with a 21 ft focal length grating.<sup>5, 3</sup> They did this by first calculating a Dirac impulse spectrum from the Dieke and Crosswhite data. They then calculated Dirac impulse spectra at temperatures from 300 K to 6000 K using Eq. 7. To these spectra, an "apparatus function" was applied to obtain spectra that

modeled spectra recorded by spectrometers having different resolving powers. Comparison of the theoretical spectrum they produced with our spectra in the region of 306 to 310 nm shows almost perfect wavelength correspondence with the sixteen peaks and five shoulders. This information is summarized in Table 2.

$$I_{nm} = I_{nm\text{Ref}} \frac{Q(T_{\text{Ref}})}{Q(T)} e^{-\left(\frac{E_n(T_{\text{Ref}} - T)}{T_{\text{Ref}} T}\right)} \quad (7)$$

Several different methods for temperature estimation were presented by Pellerin, et al. In the first method, Pellerin, et al. plotted a series of spectra of OH in the 306 to 310 nm range at 300, 3000 and 6000 K and at four different apparatus function values of 0.025, 0.050, 0.075 and 0.100 nm. The temperature was to be estimated by visual comparison of the displayed spectra to the one recorded on our spectrometer. Our spectrum appeared to be closest to the spectra having an apparatus function of 0.100 nm. Visual interpolation indicated our plume temperature to be greater than 3000 K. The second method involved taking ratios of spectral band maxima in the 306.0 to 310.0 nm range and comparing the ratios with their calibration graphs using the appropriate apparatus function. Table 3 shows the temperature estimates from taking the ratios of intensities of three sets of band heads. Two of the ratios indicated the temperatures in our rocket plumes were in the 3000 K range. A third ratio predicted that the temperatures were all above 6000 K. The final method we used from the Pellerin, et al. paper was for spectrometers with poor resolution (apparatus function greater than 0.100 nm). This method involved using the ratio of group maximum intensities for Group 1 (306.3 – 307.3 and Group 2 (307.6 – 308.8 nm). We estimated our temperature to be  $3000 \pm 100$  K from their Figure 5.

Charles de Izarra further developed the procedures given by Pellerin, et al. He also used the Dieke and Crosswhite data to produce Dirac impulse spectra at temperatures from 600 to 9000 K in 200 K steps. In his calculations, de Izarra made the simplification that the ratio of partition functions of  $Q_{\text{Ref}}(T)/Q(T)$  was equal to unity. He again applied a Gaussian apparatus function to each spectral line to produce spectra for spectrometers having different resolutions which is described by Eq. 8. In de Izarra's paper, the apparatus function was defined

$$G(?) = \frac{2}{? \sqrt{p}} e^{-\left(\frac{(? - ?_0)^2}{(? / 2)^2}\right)} \quad (8)$$

and its relation to Pellerin, et al. was given by Eq. 9. For example, our Pellerin apparatus function,  $d = 0.100$  nm

$$d = ? \sqrt{\ln 2} \quad (9)$$

became  $? = 0.12$  nm for the de Izarra method. In the de Izarra method, the emission intensity maximum was found for  $G_0$  (306.3 – 306.5 nm),  $G_1$  (306.7 – 306.9) and  $G_{\text{Ref}}$  (309.1 nm). Then the spectral intensity ratios,  $G_0/G_{\text{Ref}}$  and  $G_1/G_{\text{Ref}}$  calculated. These two ratios were then compared with standard tables of theoretical ratios using the appropriate apparatus constant. By interpolation, temperatures were found and recorded in Table 5. Two different sets of temperatures were found corresponding to approximately 3200 and 1600 K. From these results, our assessment is that the actual rotational temperature is in the region of 3000 to 3200 K and that certain spectral bands are more reliable than others for temperature estimation. These findings are consistent with an independent study of this same PMMA fuel grain configuration by Teague, et al.<sup>6</sup> Their method used absorption spectroscopy instead of emission spectroscopy. They found temperatures between 2500 and 3200 K at measured 75 mm from the nozzle end. Their method used a Model HR-320 JY, 0.32 m, monochromator with a 2400 grooves per mm grating to record the spectra. A multi-parameter curve fitting routine was used to provide a best fit to the spectrum using temperature and number density as the independent variables.<sup>7</sup>

Another study is summarized in the graph of Figure 4 in which is plotted the area under the OH radical emission band at 306-320 nm. This was done with a standard oxygen regulator which could not be adjusted with more accuracy than plus/minus two pounds. Because the area under an emission band is directly proportional to the amount of material, the graph shows that maximum OH production occurs at about 23 psig when measurements are made at 0 mm from the fuel grain exit end. Figure 5 is a graph of the OH emission intensity as a function of

measuring distance from the exit end of the fuel grain. Under the condition of 20 psig oxygen pressure, the maximum amount of OH, indicated by intensity of emission at 306-320 nm is found at 24 mm.

## V. Conclusion

It is difficult to obtain accurate temperatures of hybrid rocket plumes. Most studies of flame temperatures use premixed homogeneous gaseous flames. By judicious use of various published flame temperature estimation methods it appears possible to measure rotational temperatures of a PMMA hybrid rocket exhaust plume. The range of temperatures estimated in this study range from 2600 to 3400 K when oxygen pressures of 5 to 30 psig were employed. We further showed that it is possible to measure the OH concentration as function of pressure and of distance from the rocket fuel grain nozzle. We will use this knowledge as the basis for future spectral studies of hybrid rocket exhaust plumes in the near-infrared using diode lasers.

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